

Thermodynamics of the unified dark fluid with fast transition

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In the so-called unified dark fluid models, the dark sector gets simplified because dark matter and dark energy are replaced by a single fluid that behaves as the former at early times and as the latter at late times. In this short paper we analyze this class of models from the thermodynamic viewpoint. While the second law of thermodynamics is satisfied, the first two derivatives of the entropies of the apparent horizon and of the energy components suffer such a sharp oscillation that doubts are raised about the soundness of this class of models.

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Homogeneous and isotropic world models, aimed at accounting both for the present era of cosmic accelerated expansion and the large scale structure, usually assume two dark components as the chief sources of the gravitational field: dark matter and dark energy. Combined, they contribute to about 95% of the current energy density; the remaining 5% is provided by baryons. Very often, both main components are assumed to interact between them and with the other ingredients of the cosmic budget (baryons, photons, etc) only gravitationally. The former characterizes for being cold and, therefore, responsible (alongside the baryon fluid) for the formation of galaxies and clusters thereof. The latter, distinguishes itself for possessing a huge negative pressure (of the order of its energy density) that drives the accelerated expansion, and for clustering very weakly -possibly at the largest accessible scales only. The first one dominates the expansion at early times ($z > 1$); the latter, at late times (at redshifts below unity). So, in this scenario, each dark component has a well-defined and separate role in shaping our present Universe.

Nevertheless, the possibility was raised that both components are simply manifestations of a single entity that at high redshifts would behave as a pressureless fluid and at low redshifts as a cosmological constant. Well-known examples are the Chaplygin gas model [1] and its generalizations, see e.g. [2]. In these, the cosmic equation of state (EoS), i.e., the ratio between the pressure and the energy density, $w = p/\rho$, gently evolves from zero (the w value corresponding to cold matter) to -1 , typical of the quantum vacuum. In principle, this is an attractive scenario because it kills two birds with a single stone. Regrettably, it fails at the perturbative level as the corresponding matter power spectrum is at variance with observation [3] and the integrated Sachs-Wolfe effect significantly departs from the one predicted by the concordance Λ CDM model -see however [4]. In a related class of models -collectively known as “unified dark fluid” (UDF) models- that apparently are not afflicted by these problems, the transition from the Einstein-de Sitter regime to the accelerated phase occurs rather quickly; see [5–8] and references therein. Since in the accelerated phase cosmic structures essentially stops growing, UDF models feature a longer matter dominated era than the Λ CDM model and unified models with a slow transition. This may help tell apart both class of models.

Obviously, viable cosmological models, in addition to passing the observational tests, must not run into conflict with well-known physics. The target of this brief paper is to explore whether UDF models, based on general relativity, comply with the second law of thermodynamics. The latter, when applied to systems that present a horizon (as is the case of the said models), must take into account the entropy of matter and fields within the horizon plus the entropy of the horizon itself, which is proportional to its area. It arises in a natural way because the horizon prevents the observer from seeing what lies beyond it. Here, for the sake of conciseness, we shall focus on the model of Ref. [8] which, we believe, summarizes fairly well the class of UDF models.

As the cosmological horizon we shall take the apparent horizon, the marginally trapped surface with vanishing expansion, since -by contrast to other possible choices- the laws of thermodynamics are fulfilled on it [9]. Its radius

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and entropy are given by [10] $\tilde{r}_H = (H^2 + ka^{-2})^{-1/2}$ and

$$S_H = \frac{k_B}{\ell_{pl}^2} \frac{\pi}{H^2 + k a^{-2}}, \quad (1)$$

respectively. Here, and throughout, ℓ_{pl} and k denote the Planck's length and the spatial curvature index.

The second law of thermodynamics simply formalizes the empirical fact that macroscopic systems spontaneously tend to thermodynamic equilibrium. In essence it asserts that the entropy, S of isolated systems can never decrease (i.e., $S' \geq 0$), and that eventually it tends to a maximum (i.e., $S'' \leq 0$) compatible with the constraints of the system [11]. Here the prime means derivative with respect to the relevant variable.¹

Before going any further, we remark that given the strong connection between gravity and thermodynamics [12–15] it is natural to expect that the Universe behaves as a normal thermodynamic system; i.e., that it approaches a state of maximum entropy in the long run [16, 17].

UDF models usually enter the following energy components: radiation, baryons and the unified fluid (subscripts b , r and u , respectively). As mentioned above, the latter plays the role of cold dark matter at early times and dark energy later on. Thus, the entropy of the Universe is contributed to by the entropy of these plus that of the horizon,

$$S = S_r + S_b + S_u + S_H. \quad (2)$$

It must never decrease and it must be concave ($S'' < 0$) when $a \rightarrow \infty$.

On the other hand, the Einstein field equations, assuming a spatially flat metric ($k = 0$), read

$$H^2 = H_0^2 \left[\frac{\Omega_{b0}}{a^3} + \frac{\Omega_{r0}}{a^4} + \Omega_u \right], \quad (3)$$

and

$$\frac{H'}{H^2} = -\frac{3}{2aH} \left(1 + \sum_i w_i \Omega_i \right), \quad (i = b, r, u) \quad (4)$$

where $w_r = 1/3$ and $w_b = 0$. As usual, the Ω_i quantities denote the fractional density ($\Omega_i = \rho_i / \sum_i \rho_i$) of the corresponding component, and a zero subindex attached to a quantity indicates that it is to be evaluated at the present time.

Here we consider a UDF model in which the transition is parametrized by the EoS [7, 8]

$$w_u = -\frac{1}{2} \left[\tanh \left(\frac{a - a_t}{\beta} \right) + 1 \right]. \quad (5)$$

It contains two positive-definite, but otherwise free parameters: a_t , the scale factor at which the transition takes place, and β that gauges how quickly the transition proceeds (the smaller β , the faster the transition). For $a \ll a_t$ one has $0 \geq w_u \gg -1$, as illustrated in Fig. 1 of [8]; in fact, the faster the transition, the smaller the ratio $-a_t/\beta$ and the hyperbolic tangent approaches -1 . On the other side, when $a \gg a_t$, the hyperbolic tangent tends to 1 and $w_u \simeq -1$.

The conservation equation for the unified fluid, $\rho'_u = -3(1 + w_u)/a$, leads, after integration, to

$$\Omega_u(a) = (1 - \Omega_{b0} - \Omega_{r0}) \exp \left\{ -3 \int_0^a \frac{[1 + w_u(x)]}{x} dx \right\}, \quad (6)$$

¹ Sometimes one come across mutilated versions of this law that leave aside the above condition on S'' . While it works well for many practical purposes, it is insufficient in general. Otherwise, one would witness systems with an always increasing entropy but never achieving equilibrium, something in sharp contrast with daily experience.

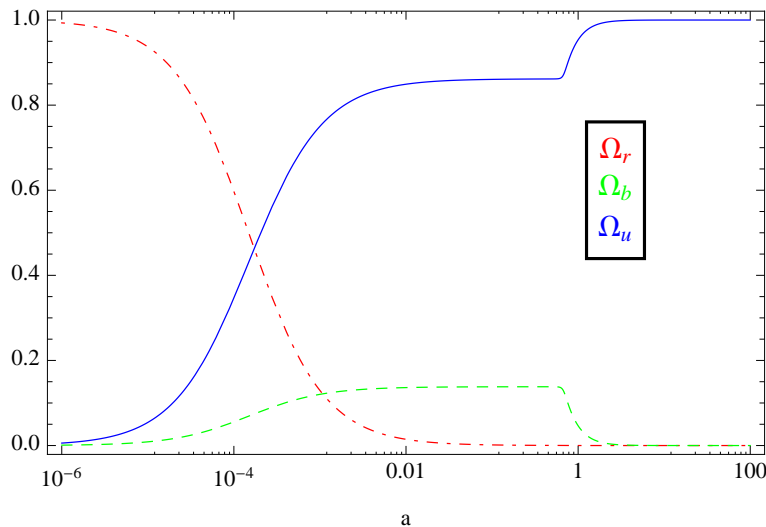


FIG. 1: Evolution of the density parameters Ω_i of the UDF fast transition model of Ref. [8]. Solid, dot-dashed, and dashed lines denote Ω_u , Ω_r and Ω_b , respectively. In plotting this we used the best fit values derived in [8]; namely: $\Omega_{r0} = 5 \times 10^{-5}$, $\Omega_{b0} = 0.0465$, $H_0 = 2 \times 10^{-18} \text{ s}^{-1}$, $a_t = 0.674$ and $\beta = 0.249$.

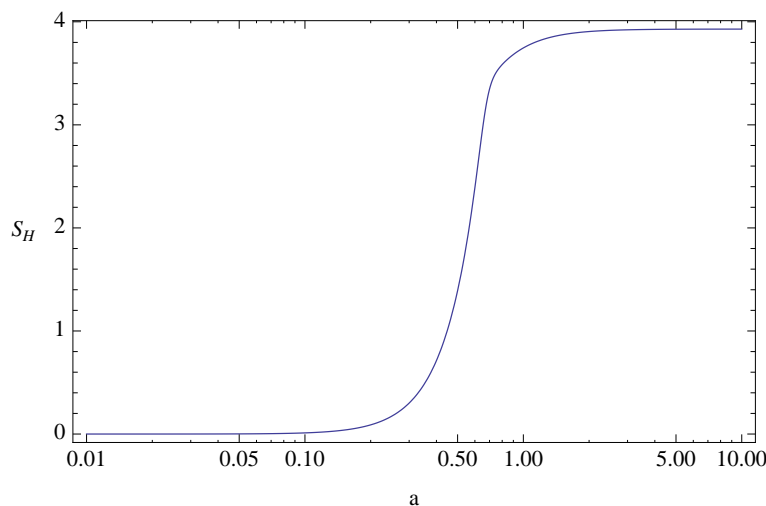


FIG. 2: Evolution of the entropy of the apparent horizon in terms of the scale factor. In plotting the graph we used the same values employed in the previous figure. The numerical values in the vertical axis are to be multiplied by the factor $k_B \ell_{pl}^{-2} \simeq 5.4 \times 10^{49} \text{ erg/Kelvin}$.

where, without loss of generality, we have set a_0 to unity. Figure 1 plots the evolution of the fractional densities of all components (baryons, radiation, and the dark fluid) using the best fit values found in [8]. The sudden increase (decrease) of Ω_u (Ω_b) at $a_t \sim 0.6$ is a distinguishing feature of the model.

The evolution of the entropy of the apparent horizon is shown in Fig. 2. In conformity with the expression

$$S'_H = \frac{dS_H}{da} \propto \frac{3}{aH^2} \left(1 + \sum_i w_i \Omega_i \right) \geq 0 \quad (7)$$

[which was obtained with the help of (4)], it never diminishes. On the other hand, the curvature of the graph changes from positive to negative values about the transition scale factor, a_t ; i.e., when the dark fluid begins to fully dominate the expansion.

To get the second derivative of the entropy of the apparent horizon we first express the derivative of the fractional densities of the various components in terms of the original quantities, i.e.,

$$\Omega'_i = \frac{3}{a} \Omega_i \left[\left(\sum_i w_i \Omega_i \right) - w_i \right], \quad (8)$$

and obtain

$$S''_H \propto \frac{3}{a^2 H^2} \left[a \sum_i w'_i \Omega_i + 6 \left(\sum_i w_i \Omega_i \right)^2 + \sum_i w_i \Omega_i (5 - 3w_i) + 2 \right]. \quad (9)$$

As readily seen, $S''_H(a \rightarrow \infty) \rightarrow 0$, is in agreement with the graph of S_H of Fig. 2.

From Eqs. (7) and (9) we infer that the apparent horizon of the UDF model of Ref. [8] (and, in general, of every reasonable UDF model) satisfies the second law of thermodynamics. However, it would be too premature to jump to the conclusion that this guarantees the fulfillment of the second law for the Universe itself. It could happen that at some stage of the expansion the said law would get violated by one or more fluid components and a breakdown of the second law would be induced. Nevertheless, given the multiplicative factor $k_B \ell_{pl}^{-2}$ in the expression for S_H , it is natural to expect that, indeed, the entropy of the horizon dominates over that of every component. In fact, this is the case by a factor of 18 orders of magnitude in the present Universe [18]. At any rate, it is safer to investigate the behavior of the two first derivatives of the fluid components to check whether the total entropy, given by the right-hand side of (2), comply with the said law.

The variation of the entropy of the radiation fluid and the unified fluid component follows from Gibbs's law

$$T_k dS_k = d \left(\rho_k \frac{4\pi \tilde{r}_H^3}{3} \right) + p_k d \left(\frac{4\pi \tilde{r}_H^3}{3} \right), \quad (k = r, u) \quad (10)$$

where T_k denotes the fluid temperature, which is always positive definite. With the help of (3) and (4) it can be recast as

$$T_k S'_k = \frac{3\Omega_k(1 + w_k)}{4GaH} [1 + 3(w_r \Omega_r + w_u \Omega_u)]. \quad (11)$$

Bearing in mind the expression for w_u [Eq. (5)] and Fig. 1, one realizes that, except when the scale factor is small, S'_r and S'_u are bound to be negative.

From Gibbs equation and the condition that dS_k be a differential, one obtains $d \ln T_k / d \ln a = -3w_k$. Consequently,

$$T_r = T_{r0} a^{-1} \quad \text{and} \quad T_u = T_{u0} \exp \left[-3 \int_1^a \frac{w_u(x)}{x} dx \right]. \quad (12)$$

Because the baryon fluid behaves essentially as dust, its temperature vanishes and Gibbs's equation cannot be employed to calculate the evolution of its entropy. Here we resort to the procedure followed in [16]. Consider that every dust particle contributes to the entropy of this component by a given bit, say k_B . Hence, within the apparent horizon we will have $S_b = k_B N$, where $N = n(4\pi/3)\tilde{r}_H^3$ denotes the number of particles there and $n = n_0 a^{-3}$ is the number density of dust particles. Thus, with the help of (4) we get

$$S'_b = \frac{2\pi n_0 k_B}{a^4 H^3} [1 + 3(w_r \Omega_r + w_u \Omega_u)]. \quad (13)$$

Again, it is apparent that from some scale factor onward S'_b will be negative.

Figure 3 depicts the evolution of dS_i/da ($i = H, u, r, b$) for the UDF model (left panel) and the concordance Λ CDM model (right panel). The latter is shown for the sake of comparison.

Likewise, Fig. 4 depicts the evolution of S''_H for the UDF model (left panel) and the concordance Λ CDM model (right panel). The corresponding second derivatives of the fluid components of either model are not shown because they fall by many magnitude orders below S''_H , whereby their graphs would essentially overlap the horizontal axis.

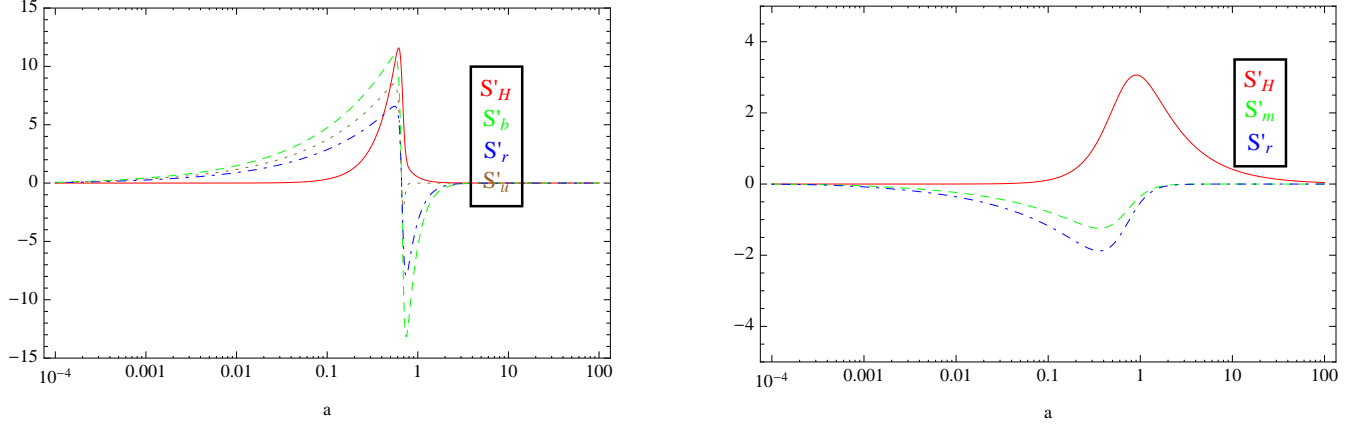


FIG. 3: Left panel: Evolution of dS/da for the apparent horizon and the energy components of the UDF model. The scales for S'_H , S'_u , S'_r and S'_b , should be multiplied by a factor of 10^{99} , 10^{89} , 10^{77} , and 10^{50} , respectively. Note that, S'_u , S'_r and S'_b becomes negative near the present epoch and remains so forever. However, S'_r is positive all the same because S'_H dominates by a huge margin. In plotting the graphs we used the same values as in Fig. 1. Right panel: *Idem* for the Λ CDM model. The subscript m stands for the pressureless energy components, baryons plus cold dark matter. In plotting the graphs we used $\Omega_{r0} = 5 \times 10^{-5}$, $\Omega_{m0} = 0.27$ and $\Omega_{\Lambda0} = 1 - \Omega_{r0} - \Omega_{m0}$.

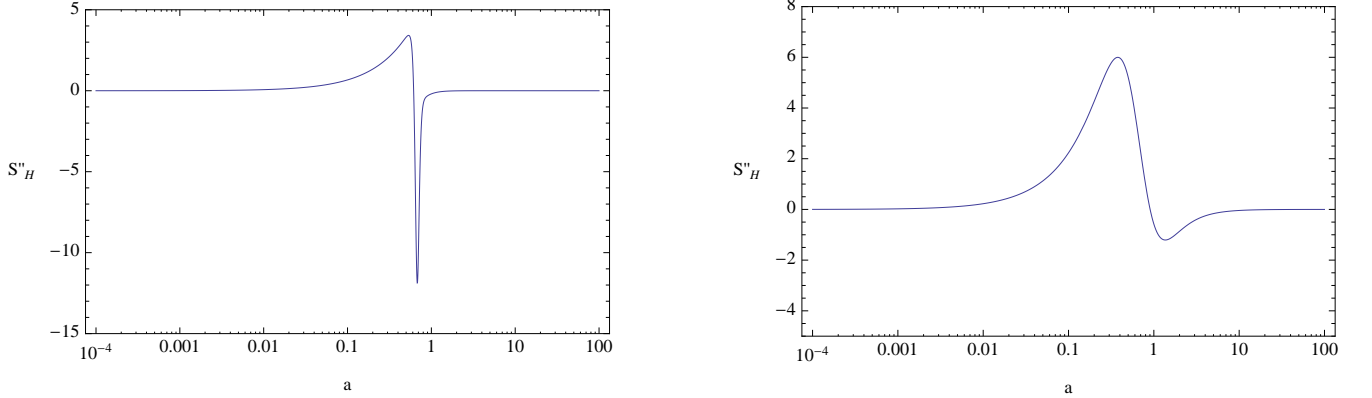


FIG. 4: Left panel: d^2S_H/da^2 vs. the scale factor for the UDF model. In drawing the plot we used the values employed in Fig. 1. Right panel: The same but for the concordance Λ CDM model. In plotting this graph we used $\Omega_{r0} = 5 \times 10^{-5}$, $\Omega_{m0} = 0.27$, and $\Omega_{\Lambda0} = 1 - \Omega_{r0} - \Omega_{m0}$.

Given the overwhelming dominance of the entropy of the horizon (and its two first derivatives) over the entropies of the fluid components, it follows that the second law of thermodynamics is satisfied by the UDF model (as well as by the Λ CDM model) thanks to the behavior of S_H . Put another way, the fluid components do not by themselves satisfy the aforementioned law. If it were not by the horizon entropy neither model would comply with it. Should it be so, one would conclude that either of these models are unphysical or that the second law does not apply to cosmological scales. However, the latter conclusion would be hard to swallow in view, as mentioned above, of the close link between gravitation and thermodynamics [12–15].

The left panels of Figs. 3 and 4 show a strong and sudden oscillation in the entropy derivatives of the UDF model that starts well before the transition scale factor a_t is attained (i.e., while w_u still mimics the EoS of pressureless dark matter) and ends up shortly after the present time. By contrast, as shown in the right panels of these figures, the oscillation is much less severe in the Λ CDM model (it is of a much smaller amplitude, and grows and decays more slowly) and finishes much later (especially in S'_H).

This different behavior can be traced to the evolution of dw_u/da depicted in Fig. 5. The w'_u big dip, centered about a_t , accounts for the quick variation of the first derivative of the entropies, and the second derivative of S_H . By contrast, in the Λ CDM model (which has $w_\Lambda = -1$ and $w_m = 0$ at any scale factor), the said variations are necessarily much softer.

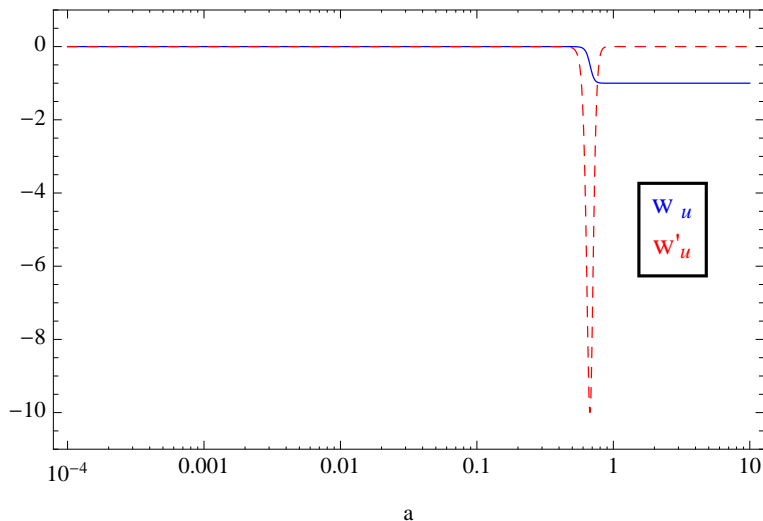


FIG. 5: The EoS of the UDF (solid line) fluid and its first derivative (dashed line) vs. the scale factor. In plotting the graphs we used $a_t = 0.674$ and $\beta = 0.249$.

In summary, on the one hand, the UDF class of models fulfill the second law of thermodynamics, i.e., its total entropy (that of the horizon plus matter and fields inside it is never decreasing and it tends to a maximum as $a \rightarrow \infty$). On the other hand, owing to the abrupt behavior of the EoS, the first and second derivatives of the entropy present a rather peculiar, sharp oscillation that casts doubts on the soundness of this class of models.

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